MECHANICS (C) UNIT 1 **TEST PAPER 7**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

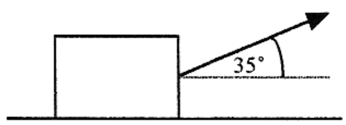
1. Briefly define the following terms used in modelling in Mechanics:
   (i) lamina, (ii) uniform rod, (iii) smooth surface, (iv) particle. [4]

2. A particle P moves in a straight line so that its velocity v ms^{-1} at time t seconds is given, for $t \geq 1$, by the formula $v = 2t + \frac{8}{t^2}$. Find the time when the acceleration of P is zero. [4]

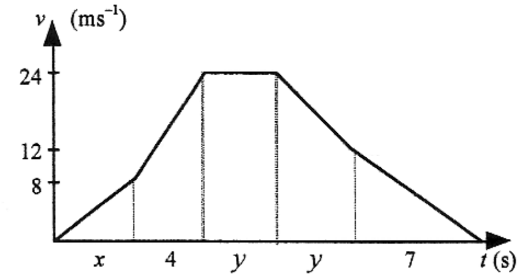
3. **F** and **G** are two forces. **F** has magnitude 15 N and acts on a bearing α, where $\alpha < 90°$ and $\tan \alpha = \frac{3}{4}$. **G** has magnitude 13 N and acts on a bearing β, where $\beta < 90°$ and $\tan \beta = \frac{12}{5}$.
   The resultant of **F** and **G** is **R**.
   Calculate the magnitude of **R** and the bearing of the direction in which **R** acts. [6]

4. Two small smooth spheres A and B, of equal radius but masses m kg and km kg respectively, where $k > 1$, move towards each other along a straight line and collide directly. Immediately before the collision, A has speed 5 ms^{-1} and B has speed 3 ms^{-1}. After the collision, the speed of B is 4 ms^{-1}.
   (i) Show that the speed of A immediately after the collision is $(7k - 5)$ ms^{-1} and deduce that the direction of A's motion is reversed. [5]
   B now receives a further impact in which the change in its momentum is mu Ns, as a result of which a second collision between it and A occurs.
   (ii) Show that $u > k(7k - 1)$. [4]

5. A string is attached to a packing case of mass 12 kg, which is at rest on a rough horizontal plane. When a force of magnitude 50 N is applied at the other end of the string, and the string makes an angle of 35° with the vertical as shown, the case is on the point of moving.



   (i) Find the coefficient of friction between the case and the plane. [5]
   The force is now increased, with the string at the same angle, and the case starts to move along the plane with constant acceleration, reaching a speed of 2 ms^{-1} after 4 seconds.
   (ii) Find the magnitude of the new force. [5]
   (iii) State any modelling assumptions you have made about the case and the string. [2]

6. 

The velocity-time graph illustrates the motion of a particle which accelerates from rest to 8 ms^{-1} in x seconds and then to 24 ms^{-1} in a further 4 seconds. It then travels at a constant speed for another y seconds before decelerating to 12 ms^{-1} over the next y seconds and then to rest in the final 7 seconds of its motion.

Given that the total distance travelled by the particle is 496 m,

(i) show that $2x + 21y = 195$. [4]

Given also that the average speed of the particle during its motion is 15·5 ms^{-1},

(ii) show that $x + 2y = 21$. [3]

Hence find

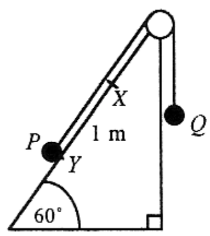
(iii) the values of x and y, [2]

(iv) the acceleration for each section of the motion. [3]

7. X and Y are two points 1 m apart on a line of greatest slope of a smooth plane inclined at 60° to the horizontal. A particle P of mass 1 kg is released from rest at X.

(i) Find the speed with which P reaches Y. [4]

P is now connected to another particle Q, of mass M kg, by a light inextensible string. The system is placed with P at Y on the plane and Q hanging vertically at the other end of the string, which passes over a small fixed pulley at the top of the plane.



The system is released from rest and P moves up the plane with acceleration $\frac{g}{5}$.

(ii) Show that $M = \frac{5\sqrt{3}+2}{8}$. [7]

State a modelling assumption that you have made about the pulley. Briefly state what would be implied if this assumption were not made. [2]

MECHANICS 1 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME

1. (i) 2-D rigid body (ii) 1-D rigid body, centre of mass at mid-pt. B1 B1
   (iii) No frictional force (iv) Mass concentrated at a point B1 B1 **4**

2. $a = 2 - 16t^{-3} = 0$ when $t^3 = 8$ $\quad t = 2$ M1 A1 M1 A1 **4**

3. Net force to east = 9 + 12 = 21 Net force to north = 12 + 5 = 17 B1 B1
   Resultant = $\sqrt{(21^2 + 17^2)} = 27{\cdot}0$ N M1 A1
   Direction (as bearing) = $\tan^{-1}(21/17) = 51{\cdot}0°$ M1 A1 **6**

4. (i) $5m - 3km = mv_A + 4km$ $\quad \div m$: $v_A = 5 - 7k, < 0$ as $k > 1$, so M1 A1 M1
   speed of $A = (7k - 5)$ ms^{-1} and direction is reversed A1 A1
   (ii) B's speed is now increased by $\frac{u}{k}$ and its direction changed, M1
   so must have $\frac{u}{k} - 4 > 7k - 5$ $\quad \frac{u}{k} > 7k - 1$ $\quad u > k(7k - 1)$ M1 A1 A1 **9**

5. (i) Resolve : $R + 50 \sin 35° = 12g$, $\quad 50 \cos 35° = \mu R$ M1 A1 A1
   $\mu(12g - 50 \sin 35°) = 50 \cos 35°$ $\quad \mu = 0{\cdot}461$ M1 A1
   (ii) Resolve : $R + F \sin 35° = 12g$, $\quad F \cos 35° - \mu R = 12a$ M1 A1
   $a = 0{\cdot}5$: $F(\cos 35° + 0{\cdot}461 \sin 35°) = 6 + 0{\cdot}461(12g)$ $\quad F = 55{\cdot}5$ B1 M1 A1
   (iii) Case = particle (does not topple); string light and inextensible B1 B1 **12**

6. (i) Total dist. = sum of areas = $4x + 64 + 24y + 18y + 42$ M1 A1
   Hence $4x + 42y + 106 = 496$ $\quad 2x + 21y = 195$ M1 A1
   (ii) Total time = $x + 2y + 11$, so $496 = 15{\cdot}5(x + 2y + 11)$ M1 A1
   $x + 2y + 11 = 32$ $\quad x + 2y = 21$ A1
   (iii) Solving simultaneously : $x = 3, y = 9$ M1 A1 (both)
   (iv) $\frac{8}{3}, 4, 0, -\frac{4}{3}, -\frac{12}{7}$ ms^{-2} B3 (–1 e.o.) **12**

7. (i) Acc = $g \sin 60° = 8{\cdot}49$ ms^{-2} $\quad v^2 = 2as = 16{\cdot}97$ $\quad v = 4{\cdot}12$ ms^{-1} M1 A1 M1 A1
   (ii) $T - g \sin 60° = a$, $\quad Mg - T = Ma$ $\quad a = \frac{g}{5}$ M1 A1 A1
   Add : $Mg - g\frac{\sqrt{3}}{2} = M\frac{g}{5} + \frac{g}{5}$ $\quad M(\frac{4g}{5}) = \frac{g}{5} + g\frac{\sqrt{3}}{2}$ M1 A1
   x 10, ÷ g : $8M = 2 + 5\sqrt{3}$ $\quad M = \frac{5\sqrt{3} + 2}{8}$. M1 A1
   Assumed pulley is smooth. If not, tensions in two sections of string B1
   are not equal B1 **13**